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Review: The prevailing mathematical modeling classifications and paradigms to support the advancement of sustainable animal production

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ABSTRACT

Mathematical modeling is typically framed as the art of reductionism of scientific knowledge into an arithmetical layout. However, most untrained people get the art of modeling wrong and end up neglecting it because modeling is not simply about writing equations and generating numbers through simulations. Models tell not only about a story; they are spoken to by the circumstances under which they are envisioned. They guide apprentice and experienced modelers to build better models by preventing known pitfalls and invalid assumptions in the virtual world and, most importantly, learn from them through simulation and identify gaps in pushing scientific knowledge further. The power of the human mind is well-documented for idealizing concepts and creating virtual reality models, and as our hypotheses grow more complicated and more complex data become available, modeling earns more noticeable footing in biological sciences. The fundamental modeling paradigms include discrete-events, dynamic systems, agent-based (**AB**), and system dynamics (**SD**). The source of knowledge is the most critical step in the model-building process regardless of the paradigm, and the necessary expertise includes (a) clear and concise mental concepts acquired through different ways that provide the fundamental structure and expected behaviors of the model and (b) numerical data necessary for statistical analysis, not for building the model. The unreasonable effectiveness of models to grow scientific learning and knowledge in sciences arise because different researchers would model the same problem differently, given their knowledge and experiential background, leading to choosing different variables and model structures. Secondly, different researchers might use different paradigms and even unalike mathematics to resolve the same problem; thus, model needs are intrinsic to their perceived assumptions and structures. Thirdly, models evolve as the scientific community knowledge accumulates and matures over time, hopefully resulting in improved modeling efforts; thus, the perfect model is fictional. Some paradigms are most appropriate for macro, high abstraction with less detailed-oriented scenarios, while others are most suitable for micro, low abstraction with higher detailed-oriented strategies. Modern hybridization aggregating artificial intelligence (**AI**) to mathematical models can become the next technological wave in modeling. AI can be an integral part of the SD/AB models and, before long, write the model code by itself. Success and failures in model building are more related to the ability of the researcher to interpret the data and understand the underlying principles and mechanisms to formulate the correct relationship among variables rather than profound mathematical knowledge.

Implications

Fundamental education in mathematical modeling is needed to regress the aversion to mathematics by biologists. The three most commonly used paradigms for modeling problems are discrete-events, agent-based, and system dynamics, but hybrid paradigms exist. The unreasonable effectiveness of mathematical models might be related to the fact that different researchers would model the same problem differently, given their scientific knowledge and experiential background, leading to choosing different variables and model structures. Mental concepts do not need data; they require ideas of how variables are connected because data will become available in one way or another if the concept is sound.

Introduction

Mathematical modeling is, in its essence, the art of reductionism of scientific knowledge into an arithmetical layout. It fuses data with pre-established concepts and visceral understandings of the scientific knowledge gained on the subject of interest during

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the learning process. Therefore, mathematical models translate our perceptions of real-life facts and observations into virtual representations using mathematical formulations (Tedeschi, 2019). Subsequently, sensitivity and stress analyses, using independent data, validate and evaluate the behavior and predictability of the model, i.e., its adequacy for a purpose (Tedeschi, 2006), while confronting our hypothesis and concepts about reality. Suitable and robust statistical designs, methods, and analyses are critical to mathematical models' development and evaluation phases and concept formation during the learning process, usually of four kinds: experiential, presentational, propositional, and practical (Heron, 2009). In the first instance, statistical mediation is needed partly because of inherited, unknown, and uncontrollable errors and mistakes associated with the researcher's data-gathering phase. Secondly, the human mind achieves astonishing feats at associating, recognizing, and comprehending complex concepts while forming hypotheses and ideas behind abstract experiences (National Research Council, 2000). However, although the mental representation of numerical quantities may constitute a distinct format for thought (Friedenberg and Silverman, 2006), the human mind fails miserably to process numerical calculations as the number of variables and their interactions increase (Perlovsky and Ilin, 2012).

The power of the human mind is well-documented for idealizing concepts and creating virtual reality models. Nonetheless, juggling too many balls at the same time can be challenging, if not impracticable. As our hypotheses grow more complicated and interconnected as the availability of more complex data (i.e., big data) expands, mathematical modeling earns more noticeable footing in biology (Vittadello and Stumpf, 2022), including agriculture (Thornley and France, 2007) and, in particular, animal science (Tedeschi et al., 2005; France and Kebreab, 2008; Tedeschi and Menendez, 2020). Therefore, modeling biological science is inherently more complicated than physical science, partly because biological systems rely almost exclusively on experimental trials and observational data, which are, by design, reductionisms of reality conducted under a controlled environment. In biology, a controlled environment has to be interpreted with some caution, meaning, as much as possible, a user-controlled environment within reach, given the experimental conditions. Tedeschi (2022a) presented a preliminary version of this document. This paper aims to highlight prevailing mathematical modeling classifications and paradigms in discussing and exemplifying model hybridization to support the advancement of sustainable animal production.

Modeling natural sciences

Unfortunately, biological and physical modeling differences do not stop with their natural characteristics; they are deeply influenced by other factors (Vittadello and Stumpf, 2022). As an example, May (2004) described that Charles Darwin would have benefited from the mathematical importance of Gregor Mendel's particulate nature of inheritance to understand better that simply blending maternal and paternal characteristics does not alter the gene frequency in the population. Such a feat would require external *forces* to be exerted upon the population, for example, migration, mutation, and *controlled* selection, to list a few. The differences between modeling biology and physics are deeply rooted in the individual's education. Besides the intrinsic aversion to mathematics by *biologists*, often attributed to a poorly taught course or preconceived idea that biology and mathematics do not blend, biologists are more pragmatic than physicists. Physicists want to understand the theoretical fundaments of the problem, whereas biologists want to observe, conclude, and apply. Vittadello and Stumpf (2022) contended that physicists rely on principles to build their models, whereas biologists use heuristic models, i.e., learning by trial and error under a controlled environment, to gain experience. Additionally, the curriculum in physical sciences relies heavily on mathematics and calculus, whereas in biological sciences, it is based on qualitative aspects of life, biochemistry and metabolism, and statistical methods to infer observational and experimental data. Exceptions exist for those in biological sciences with interest in data analytics and modeling reasoning.

Therefore, mathematical modeling principles might be more straightforward in physics than biology. The criticism about using mathematics in natural sciences, including physics and biology, has reached a higher level of discussions about its usefulness (or lack of) (Wigner, 1960) that resulted in several reactions from great minds in different science fields (Hamming, 1980; Lesk, 2000) to ponder about the importance of philosophy of life, existence, and mathematics. Stochasticity (i.e., randomness), perhaps more in biology than in physics, affects how mathematical modeling is perceived by their respective scientific community, given its impact on the predictability of the outcome. However, associating the frustrations and lack of trust in mathematical modeling predictability is digressing from the goal of modeling. Stochasticity is inherited from the system or problem, and it cannot alone be blamed for the failures in mathematical modeling. Other factors might be more influential to the dissuasion of the applicability and usefulness of mathematics in modeling. Hamming (1980) proposed four factors that might be associated with the unreasonable effectiveness of mathematics in natural sciences: we see what we look for, we select the kind of mathematics to use, science answers comparatively few problems, and the evolution of man provided the model. Essentially, there are three main reasons why models may (or may not) be effective conveyers of scientific learning and knowledge in biological sciences. Firstly, different researchers would model the same problem differently, given their scientific knowledge and experiential background, leading to choosing different variables and model structures (e.g., feedback loops and feedforward structures); thus, no two models are alike. Secondly, different researchers might use different modeling paradigms and even unalike mathematics to resolve the same problem; thus, model needs are intrinsic to their perceived assumptions and structures. Lastly, models evolve as the scientific understanding by the scientific community accumulates and matures over time, hopefully resulting in improvements in the modeling efforts; thus, the *perfect* model will never be. Therefore, success and failures in the mathematical modeling of natural sciences, including biology, are more likely directly related to the ability of the researcher to interpret the data (i.e., behaviors, events, outcomes) and understand the underlying principles and mechanisms to guide the correct mathematical formulation. After all, one can only build a mathematical model based on their grasp of the problem and accumulated knowledge. Another reason for model failure is the unnecessary complexity of the mathematical model that overwhelm even experienced users, making them lose sight of the forest for the trees (Tedeschi, 2019). Thus, successful model development might come from, for example, three-quarters creativity and only one-quarter mathematical formulation. Whether mathematics and mathematical modeling have a place in biology is an ongoing discussion that has drawn supporters and dissenters from different fields of science (Wigner, 1960; Cohen, 2004; Tegmark, 2008; Abbott, 2013). Mathematical modeling is needed to understand the very reason for the existence of biological sciences. Given the combinatorial possibilities of the multitude of factors and variables through randomness and all possible solutions to the problem, mathematical models can assist with interpreting the data and shedding light on the occurrences of events observed in biological sciences through experimentation; thus, mathematical modeling and biological sciences should walk hand-in-hand. Nevertheless, the question remains, how do we build mathematical models in biological sciences?

Modeling paradigms

The fundamental modeling paradigms include discrete-events (**DE**), dynamic systems (**DS**), agent-based (**AB**) or individual-based, and system dynamics (**SD**) modeling (Borshchev and Filippov, 2004; Tedeschi, 2019). Both DS and SD modeling are also called equation-based modeling (**EBM**); thus, models developed using these approaches are evaluated, whereas AB modeling is based on emulation (i.e., agents that emulate reality through specific attributes and attitudes). The DE and AB modeling rely exclusively on stochasticity to create events where the system's overall behavior changes. The three most commonly used paradigms for modelling business, socioeconomic, and agricultural problems are DE, AB, and SD. However, hybrid modeling paradigms also exist, such as discrete dynamical modeling (Sandefur, 1991; 1993), and newer hybridization are being proposed, especially those aggregating artificial intelligence (**AI**) as big data become increasingly available to researchers (Tedeschi, 2022b). The combination of different modeling paradigms occurs when different levels of analysis are required, such as temporal (daily vs decades time horizon) and spatial scales (micro vs macro level) of social and ecological dynamics (Martin and Schlüter, 2015).

Fig. 1 depicts the ideal association between model paradigms and problem types based on levels of abstraction, aggregation, and details using the black to gray to white analogy. The black box type refers to mathematical models in which input and output variables are known, but the system or the problem is largely unknown and is characterized by a high level of abstraction and aggregation with a low level of details (i.e., information), most of which must usually be inferred using inverse problem (Karplus, 2003; Borshchev and Filippov, 2004). On the other hand, the white box-type models represent those systems or problems with low levels of abstraction and aggregation, containing high levels of details, thus, being more deductive and less inductive (Karplus, 2003; Borshchev and Filippov, 2004). White box-type models usually present more satisfactory predictive performance than black box-type ones. Most control systems, electric circuits, and traffic models are deemed white box models, whereas most socio-politico-environmental systems or problems are on the other end of the grayness spectrum, closer to the black box definition. Most systems or problems based on chemical reactions and reaction rates are in the middle of the grayness scale of abstraction (Fig. 1), and they comprise most biological systems.

Classification of mathematical models

Classification of mathematical models is more a matter of opinion on whether some aspects of mathematical structure are used or even to which extent it is incorporated in the model, especially when the model classification does not have a clear-cut design. Model classification can become a point of contention among modelers because some might prefer one type over another or even have different perceptions about the superiority of the model predictability if a specific class is used. Mathematical models are classified in many different ways depending on their developmental programming context, style, and scope, including *optimization* (linear vs non-linear programming), *application* (descriptive or elucidative vs predictive or prescriptive), *time representation* (static or steady-state vs dynamic), *time continuity* (discrete vs continuous), *calculation mode* (deterministic vs stochastic or probabilistic), *nature* (empirical vs mechanistic or theoretical or rational), or *space* (homogeneous vs heterogeneous) (Haefner, 2005; Tedeschi and Menendez, 2020). Mechanistic models are called process-oriented or concept-driven models, whereas AI models are called data-driven models. In general, all modeling paradigms can adopt different modeling contexts, styles, and scopes, but some might be more appropriate to specific paradigms. For instance, DE and AB rely almost exclusively on stochastic elements, whereas DS and SD are frequently developed with deterministic, continuous, and dynamic attributes.

Discipline maturity usually sets the appropriate use of models based on the degree of uncertainty of the problem or systems. The incorrect use of a model for a more rigorous purpose than appropriate can be misleading or dangerous (Haefner, 2005). Generally, the model's intended use, such as to describe, predict, and explain behavior, partially pre-establishes its classification (Tedeschi and Fox, 2020). For instance, descriptive and predictive

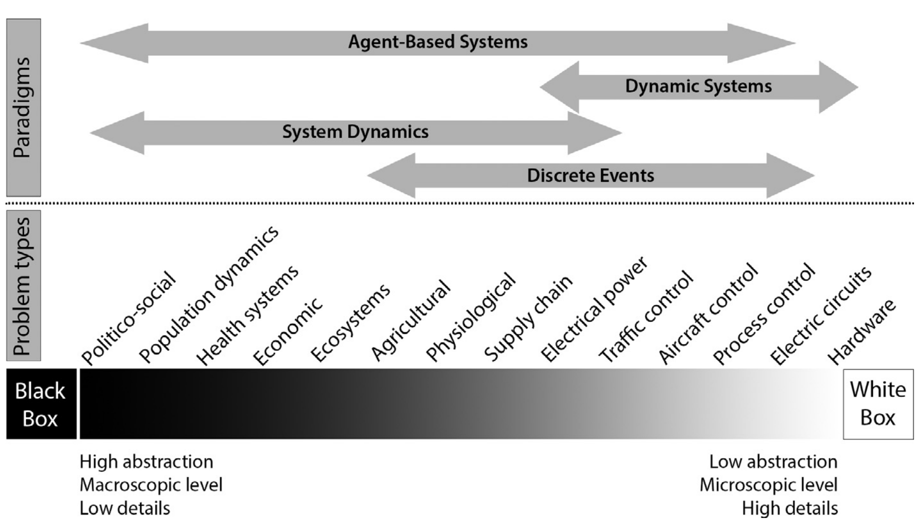

Fig. 1. Schematic representation of the association between model paradigms and problem types based on levels of abstraction, aggregation, and details. The shades of gray (gradient) indicate the level of knowledge of the systems or problem from the black box (primarily unknown) to the white box (mostly known). Based on Karplus (2003) and Borshchev and Filippov (2004).

models are usually static and empirical, whereas explanatory or mechanistic models may require a dynamic approach. Differential equations, which may be solved analytically or numerically, are implicitly or explicitly used with dynamic models.

Deterministic vs stochastic modeling

Among the modeling classification types described above, deterministic and stochastic modeling deserve particular attention given their importance for modeling sustainability and climate-related issues; the others are intuitive and are comprehensively discussed in modeling textbooks (Haefner, 2005). Heinz (2011) detailedly compared deterministic and stochastic modeling. Models classified as purely deterministic will always yield the exact solution for the same set of inputs. That means the model depends exclusively on initial values and calculations performed during the simulation process, as long as similar assumptions and decisions are constant between model executions—no surprises will befall. In contrast, critical stochastic elements must be adopted when variability becomes the main focus of the modeling effort. In climate models (Latif, 2022), the need to understand the variability and which variables are associated with the variability of climate models has boosted the interest in stochastic modeling in the last hundred years. From the fundamental discovery by Svante Arrhenius in the 1890s that doubling the concentration of CO_2 in the atmosphere would increase the atmospheric temperature by 5 °C (Arrhenius, 1896) to the latest development by Syukuro Manabe's laboratory in the 1980s of three-dimensional modeling to study the anthropogenic influence on climate changes (Stouffer and Manabe, 2017), stochasticity has been critical to understanding climatic variability caused by internal forces (Hasselmann, 1976; Frankignoul and Hasselmann, 1977). A crucial limiting factor in stochastic modeling is that the computational challenge increases drastically as finer resolutions (smaller scale or shorter timesteps) are needed to capture extreme events (or variables) that can tremendously influence the outcome. In modern climate models, the resolution is somewhat flexible/variable, which means a smaller resolution is used when needed for certain aspects or processes, but a larger resolution remains for other less critical modeling aspects or processes with minor importance (Latif, 2022). A similar strategy could be adopted in biological sciences, especially animal science modeling. For instance, finer resolutions such as cell-level processes and minute-span timesteps could be used for hormonal/endocrinological modeling, tissue-level processes and day-span timesteps for production modeling, or region-level processes and year-span timesteps for epidemiological modeling.

Essential elements in building mathematical models

A good modeling praxis is consistently identifying the research problem before selecting the best model paradigm/method to solve it. Regardless of the chosen modeling paradigm, to build a mathematical model, the source of knowledge is the most critical step. Forrester (1980) discussed the necessary expertise to build a model. The foremost expertise is possessing adequate mental concepts that might have been acquired in different ways (e.g., observation, experience) and will provide the structure and behavior of the model (Forrester, 1980). A clear definition of the problem being investigated and its boundaries (spatial and temporal) are critical to assist in identifying vital variables and planning for possible relationships among them. The expected reference modes of vital variables (i.e., their trajectory over time) are valuable to help identify existing archetypes (exemplified later) to be used as the foundational blocks of the mathematical model. The type of data and expertise needed will assist in organizing meetings with experts or other stakeholders who have invested interest in the mathematical model to gather information and learn more about unknown specifics of the mental model. The second most essential expertise is identifying published material highlighting existing concepts and abstractions about reality (Forrester, 1980). The written material is critical because it provides different thoughts about concepts and relationships of interest that will guide the modeler in selecting the variables and essential parameters in the modeling and simulation processes. The third essential expertise is acquiring numerical data, which is the least essential element in building a mathematical model (Forrester, 1980), but helpful in performing data analytics, including descriptive and advanced statistical analyses. Different approaches exist to describe the modeling process (Haefner, 1996). Fig. 2 depicts a generalized representation of a model-building process, including the model formulation, development, calibration, evaluation, validation, documentation, and deployment phases. Additional descriptions for each phase are discussed by Fishman (2001), Hannon and Ruth (2001), Dym (2004), Haefner (2005), and Law (2007), among many others.

Discrete-events modeling

Discrete-events modeling (**DEM**) depends on the change of the value or state of variables of interest at discrete points rather than continuously with time (Fishman, 2001; Law, 2007). The simulation is *on pause* until an event occurs; however, when multiple threads are being modeled, some parts of the model might be *on pause,* waiting for a specific change to occur while other parts of the model are being executed. A typical example of DEM is the customers queueing at a cashier in a grocery store or at a teller in a bank until they are helped. Similarly, the time it takes for animals to drink water at a water trough depends on water availability, water drinking speed, how many animals drank before, and water valve refilling speed, to list a few. Others might be interested in knowing the mean waiting time to drink water, the probability of waiting in the queue, the probability of waiting more than twenty minutes, or the probability of an arriving animal not waiting in the queue. DEM has been adopted to model complex systems in many fields, including engineering, health, management, mathematics, military, social, telecommunications, and transportation (Fishman, 2001). Although DE and AB modeling is based on stochasticity and event-driven processes, they differ in many aspects, as highlighted by Siebers et al. (2010): (a) DEM is a process-oriented (top-down approach) focusing on the system, whereas AB modeling is an individual-based (bottom-up approach) focusing on the entities and their interactions; (b) DEM has centralized thread control of the events, whereas, in AB models, the entities control the thread of events (decentralized); (c) DEM uses a queue, whereas AB models have no concept of a queue; and (d) DEM entities flow through a system (macro behavior is sought), whereas AB modeling entities do not have a flow, and the micro-behavior of each entity creates the system behavior of the system at a macro level.

Dynamic systems modeling

Dynamic systems modeling (**DSM**) consists of a set of equations (usually ordinary differential equations) that are evaluated. Borshchev and Filippov (2004) indicated that DSM, which is heavily adopted by mechanical, electrical, and chemical engineering disciplines, is the ancestor of SD modeling. Many publications have documented DSM exhaustively (Hannon and Ruth, 1997; Ruth and Hannon, 1997; West and Harrison, 1997; Hargrove, 1998; Feurzeig and Roberts, 1999; Deaton and Winebrake, 2000; Robinson, 2001; Costanza and Voinov, 2004; Ellner, 2006; Hannon and Ruth, 2009). In animal science, it is typical to use DSM for compartmental modeling (Dhanoa et al., 1985; France et al., 2005; Crompton et al., 2008). In essence, DS and SD modeling uses the same

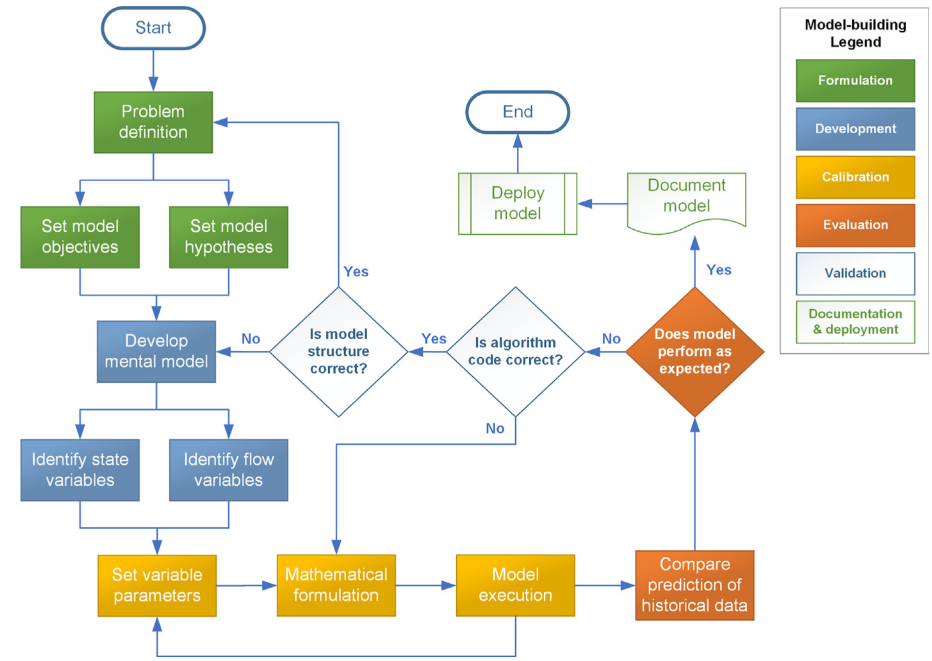

Fig. 2. A general sketch showing the essential elements and phases (model formulation, development, calibration, evaluation, validation, and documentation and deployment) during a model-building process.

mathematical formulation, notation, and even identical sketches of boxes to denominate state, level, or stock variables (unit) and arrows to represent rate or flow variables (unit/time). The critical difference between DS and SD is the methodology used in building the model: DS is usually based on *known and acceptable* mathematical preformulations such as Michaelis-Menten for enzymatic saturation kinetics, whereas the SD society advocates building models based on feedback loop structures that represent patterns (or behaviors) of variables of interest observed over time. Dynamic systems are more concerned with numerical precision, whereas SD endorses the overall pattern, though numerical precision can be modified using different mathematical formulations and methods for numerical integration (Runge-Kutta vs Euler).

System dynamics modeling

Jay Forrester initially conceived SD in the 1960s (Forrester, 1961, 1971, and 1973) to understand complex behaviors brought about circular, non-linear relationships in complex systems. System dynamics is a modeling paradigm deeply rooted in the systems thinking theory (Kramer and de Smit, 1977; Flood, 1990; Bawden, 1991; Senge, 2006). It is used to recognize that complex systems are integrated through feedback loops in which modifications made to a specific variable at a given time will ripple through the system (i.e., model), affecting other variables, but will eventually impact back the same variable over time because of feedback processes. The systems thinking theory has been applied to solve problems in ecology by Bernard Patten, sociology by Niklas Luhmann, architecture by Christopher Alexander, and business management by Jay Forrester (Fath, 2014).

Collectively, feedback processes, model structure (i.e., how variables are connected), delays (i.e., often caused by sequential state or stock variables), and intrinsic non-linearities dictate the dynamics of the system (i.e., model behavior) (Sterman, 2000). The basics of the SD methodology have been covered in several different ways, with many examples in animal science (Tedeschi et al., 2011; Nicholson et al., 2019; Stephens, 2021). The principles of SD methodology (problem boundary, dynamic hypothesis, feedback loop processes) facilitate its adoption for pragmatic and holistic applications, such as those found in social, managerial, ecological, and business systems with "dynamic complexity" (Senge, 2006). But, it does not exclude SD from being used in other science fields that seek to understand the dynamic changes of variables through time within systems characterized by interdependence, interaction, information feedback, and circular causality (Richardson, 1991). The SD literature is vast and still growing, and many articles about SD applications in animal agriculture have been published (Guimarães et al., 2009a, 2009b, and 2009c; Tedeschi et al., 2013; Turner et al., 2017; Menendez III and Tedeschi, 2020; Wanyoike et al., 2023). Typical applications of SD in biological sciences include the Lotka-Volterra equations for modeling predator–prey relationships (Lotka, 1956), the Bass diffusion model (Bass, 1969), and the susceptible-infectious-recovered for epidemiological model (Kermack and McKendrick, 1927), healthcare (Homer et al., 2004; Homer and Hirsch, 2006; Homer et al., 2007) and antimicrobial resistance (Homer et al., 2000; Stephens, 2021), among many others based in business and management strategies (Sterman, 2000; Morecroft, 2007).

Graphical representation of the mental model

An integral part of the model-building process is a graphical representation of the critical variables, their inter-relationships, and the feedback loops that can (a) capture the dynamic hypothesis, (b) stimulate the systems thinking process of mental models, and (c) exemplify the circular (i.e., feedback loops) and straight (i.e., feedforward) characteristics of the model structure that creates the causality being attributed to the system or problem. The causal loop diagram (**CLD**) represents the first sketch of the mental model because it contains idiosyncratic elements that can quickly convey the model's ideas (Sterman, 2000). It is essential to highlight that the goal of the CLD is to convey the main idea (i.e., mental model and perception of how variables are connected) and not all the variables required to model it, numerically speaking, effectively. The CLD is necessary to underpin the feedback loops responsible for complex dynamics. For instance, Fig. 3 depicts a CLD of a mental model showing the critical variables related to cattle production and its impact on global warming. The dynamic hypothesis is that the cattle herd (population) will achieve a stable but lower number under a global warming situation, and the posed question is how long it would take to stabilize at different intensities of global warming. Fig. 3 depicts six feedback loops: *beef consumption* (B1: cattle herd and beef consumption), *global warming* (B2: cattle herd, methane emissions, and global warming), *climate policies* (B3: global warming, climate policies, and methane emissions), *resource erosion* (B4: cattle herd, methane emissions, global warming, resources, available resources, and production incentives), *production incentives* (B5: cattle herd, required resources, available resources, and production incentives), and *cattle growth* (R1: cattle herd and calves).

As shown in Fig. 3, a positive sign in the arrowhead means that if the independent variable increases, the dependent variable would increase above what it would have been otherwise. If the independent variable decreases, the dependent variable would decrease below what it would have been otherwise. The negative sign follows the same notation, except the change direction of the dependent variable is opposite to the change direction of the independent variable. For instance, in the resource erosion feedback loop (B4) in Fig. 3, as *global warming* increases, one expects *resources* to decrease below the value that they would have been if global warming had been kept constant. In contrast, as *available resources* increase, one expects *production incentives* to increase above their value if available resources are kept constant. The mental model shown in Fig. 3 captures the impact of cattle's methane emission on global warming (B2) and the impact of climate policies (B3) to mitigate methane (e.g., feed additives, low methane emitter animals). The CLD does not imply that cattle are solely responsible for global warming; it implies that cattle contribute to global warming to a certain degree. Cattle's contribution to methane emission (on a CO_2 equivalent basis) is estimated to be less than 4% in the United States (Dillon et al., 2021; Tedeschi, 2022c; Tedeschi and Beauchemin, 2023), but adequate quantification is lacking (Tedeschi et al., 2022). The feedback loops R, B1, and B5 in Fig. 3 reconstruct the *Limits to Growth* archetype (Meadows et al., 1972; Meadows et al., 1992; Meadows et al., 2004; Meadows, 2009) but with an added feedback loop (B4) that erodes *resources* given the perceived impact of *global warming* on feedstuff production required to feed the cattle. Indeed, additional feedback loops can be added, but the question should be whether the existing structure is enough to mimic the observed and perceived behaviors.

Once the CLD is deemed sufficient and acceptable, the modeler can start developing the stock and flow diagram, i.e., the variables and structure needed to perform simulations (Sterman, 2000). Although the CLD is not intended for numerical simulations, it tells the story behind the model-building process by capturing the essential variables and their relationships. Often the modeler creates a CLD with variables that have not been measured (or cannot be measured for one reason or another) or do not have *accurate enough* values. This inconvenience should not prevent the modeler from building the CLD because, sooner or later, the unconventional variables will be researched, and values will be uncovered. After all, ditching a critical variable from a model because it has not been measured or its value is inaccurate is the most frequent incorrect way to acknowledge its importance. Sterman (1991) believes that assigning zero to a variable in the model, thus, omitting it, is the only invalid value the variable can take. There is no limitation on

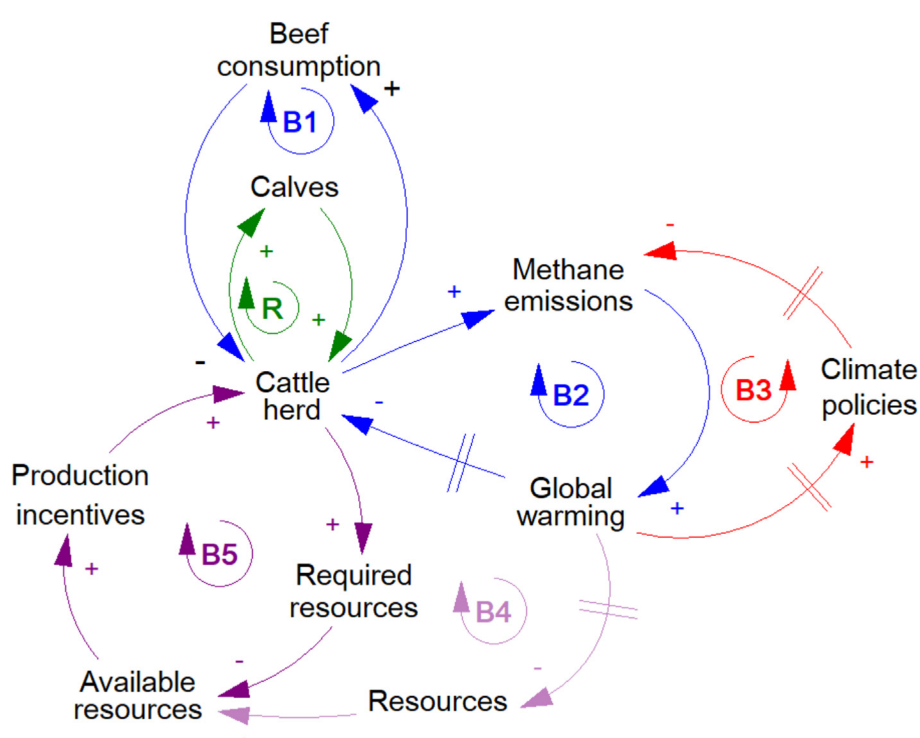

Fig. 3. A graphical representation of the mental model of some feedback loops in cattle production and its environmental impact, causing global warming. The arrows denote causal influences between two variables; the positive (+) or negative (−) sign indicates the dependent variable changes when the independent variable changes. Feedback loops can self-reinforce (R) or self-balance (B) the change of the variables in the loop. Self-reinforcing (tending to amplify changes) and self-correcting (tending to counteract and oppose changes—balancing) loops are represented by positive or "R" and negative or "B" signs, respectively, within the semi-circle arrow. The parallel lines perpendicular to the arrow line indicate a significant delay.

a model's number of variables; therefore, handicapping a model by limiting the number of variables is usually perceived as a weakness in the model development phase. Mental concepts only need the ideas of how variables are connected because data will be gathered in one way or another if the concept is sound. Tedeschi (2022a) acknowledges that removing vital variables from a model because of the perception that data do not exist will undoubtedly narrow the scientific knowledge and likely beacon the scientific community in the wrong direction of science, which might lead to an even worse situation than adding unmeasured, uncertified, non-zero values to critical variables.

Additionally, this approach pressures the scientific community to certify the essentiality of variables and validate their values. A practical example was the inclusion of fractional degradation rates to carbohydrate and protein fractions of a handful of feedstuffs to ruminant nutrition models by Sniffen et al. (1992), who initially used measured ranges for 27 grains, 21 proteinaceous feeds, and 12 forages from digestibility trials conducted before the 1990s (Tedeschi and Fox, 2020), and then *radiated out* to all feedstuffs in the feed library of the Cornell Net Carbohydrate and Protein System (Chalupa and Boston, 2003). Therefore, developing CLD to document the scientific knowledge of the time is desirable as a preliminary step toward building stock and flow diagrams. Tedeschi et al. (2014) documented the possible impacts of condensed tannins on ruminant production and built a comprehensive CLD depicting the essential variables and how they could interact within the scope of the modeling exercise. A common way to elicit CLD is to use the group model-building technique.

Group model building (Vennix, 1999; Andersen et al., 2007) is a practical approach to foster stakeholder engagement in developing interventions and policy design for sustainable management of ecological resources and socio-economical conflicts. Group model building relies heavily on building CLD through participatory discussions for data-gathering and understanding complex problems. Several studies have used CLD to develop qualitative SD models using the group model-building approach (Turner et al., 2013; Inam et al., 2015; Perrone et al., 2020; Asif et al., 2023).

Agent-based modeling

Agent-based modeling (**ABM**) is also known as multi-agent systems/modeling or individual-based modeling. Agents are autonomous computational individuals or objects with particular properties and actions built from probability distribution functions and relationships among variables of interest. The system's general behavior emerges from the agent's properties, rules, and interactions with other agents, which, in turn, influence their behaviors bounded by a context or environment (Macal and North, 2005 and 2010; Siebers et al., 2010). In essence, the individual dynamics of agents through stochastic modeling create the system's overall behavior. Therefore, by design, ABM uses mechanistic and stochastic elements, i.e., agents are confined at a micro-scale (e.g., level 0), and, through their [random] interaction, complex patterns and behaviors are expressed at a macro-scale (e.g., level 1). Thus, multi-level mechanistic simulations are possible if a hierarchical modeling approach is adopted (e.g., from molecules to cells to organs to the body). Macal and North (2009) list the following typical characteristics of an agent: (a) being self-contained, modular (i.e., has boundaries), and uniquely identifiable individual; (b) being autonomous and self-directed, i.e., functions independently in its environment and its interactions with other agents; (c) variable state over time, i.e., attributes change along with the state of the environment; and (d) being *social*, i.e., must interact with other agents that will alter its attributes; thus, its behavior as the simulation progresses. Examples of applications of ABM in animal agriculture include microbial growth dynamics (Ginovart et al., 2002; Hellweger and Bucci, 2009) and epidemiological modeling (Manjoo-Docrat, 2022).

The ABM is a powerful tool for computer simulation because its ontology is analogous to agricultural systems (Railsback and Grimm, 2011; Wilensky and Rand, 2015), making ABM a natural approach for complex systems. Additional benefits of ABM over other modeling paradigms include capturing emergent phenomena arising from the interaction among entities, providing a natural description of the system because agents are related to entities in real life, and having the flexibility to define agents programmatically (Bonabeau, 2002). The ABM allows us to develop mathematical representations from empirical observations of behavior; thus, one can conjecture about explanations of the behavior and understand the underlying principles of how these mechanisms interact. For example, understanding that behavior patterns emerge from individual interactions is a fundamental part of the ABM approach. Wilensky and Rand (2015) believe that complex systems' behavior patterns are not deterministic and derived from a central controller. For example, the goose in the front of the "V"-shape formation of goose flocks is not always the same, and this position depends on the independent behavior of each bird (i.e., agents) as there is neither a master bird that choreographs the "V"-shape formation nor such a thing as "group mind" (Macy and Willer, 2002). The "V"-shape formation emerges from the random interaction among independent geese reacting to the behavior of their closest neighbor goose. For grazing animal production situations, different combinations of forage types, management systems, and animal breeds may result in different supplementation strategies during the four seasons that might lead to more or less methane emissions and resource use, given their independent and non-mutually exclusive interactions. ABM has shined in four real-world common problems: (a) flow simulation and management such as traffic dynamics, (b) organizational simulation based on the emergent collective behavior within a particular context or level of description, (c) simulation of stock market dynamics resulting from the behavior of interacting agents, leading to emergent phenomena, and (d) diffusion simulation brought the influence people exert over others around them through social networking (Bonabeau, 2002).

Although mathematically speaking, EBM and ABM can make the exact predictions for the system's behavior, ABM can resolve controversies surrounding deterministic EBM because only ABM can also make inferences about the individual agents of the system using the stochastic approach within the same simulation run. ABM simulations can surpass dynamic EBM expectations because the agents can be programmed to change or alternate their distribution within runs, given the occurrence of specific events. For instance, animals may alter their grazing behavior within the same simulation when a forage species becomes dominant in a pasture for different reasons (e.g., climate or management). Dynamic deterministic modeling such as DS and SD rarely reflects this change at the end of the simulation, whereas ABM can incorporate it. A subterfuge for EBM is to incorporate conditional statements, such as *if-then-elseif-else*, to emulate changes during a simulation run. There are cases in which ABM is a better tool than EBM, such as when significant discrepancies exist among problem components, such as modeling two or more forage types (e.g., grass and legumes) and cows and calves sharing the same environment. In this case, while the forage types have similar properties (e.g., growth, death, nutritive quality), they are different as they behave differently depending on their interaction with climatic factors (e.g., rain, temperature, sunlight) and the level of predation by the cow and calves.

For life cycle assessment (**LCA**) analysis, ABM might be a better approach than DS or SD modeling because ABM allows tracking each agent, facilitating the collection of inputs and outputs for the LCA analysis. For instance, although cows have similar properties and actions, they differ on many variables like calf birth weight, milk yield necessary to nurture the calf, and reproduction aspects, to list a few. Each agent has a distribution for each property and action to guide their behavior, which is entirely stochastic, but the distribution parameters can be changed. Micolier et al. (2019) highlighted additional benefits of using ABM at each LCA phase, including scenario exploration, foreground inventory data collection (essential for LCA), temporal or spatial simulation dynamics, and data interpretation and communication. Only four studies out of 18 reviewed by Micolier et al. (2019) were related to agriculture, specifically the bioenergy sector. Few studies have assessed the use of ABM toward behavior-driven modeling for LCA analysis despite its innate propensity to solve the impact of agriculture products on the environment.

Another exciting feature of ABM is that it can incorporate computational advancements such as low aggregated programming language (e.g., python, java) and data storage to improve the modeling of complex problems. Moreover, agents can be added as needed to mimic the "real condition" closely. Nonetheless, computational capacity might become a limitation in simulating complex, large ABM models with many agents. In this case, a supercomputer might be required to perform the simulation, or meta-modeling (i.e., model simulation outputs used to develop another model) can be an alternative to solve the computational problem in which a group agent can be created to mimic the behavior of multiple agents, thus, reducing the number of agents of the final ABM system.

There are other limitations of ABM that modelers must be aware of before investing too much time in developing AB models. As Daly et al. (2022) discussed, the mathematical formulation, implementation, and analysis of AB models can be obscure, and lack transparency as the equations are not readily available. Such obfuscation might limit critical assessment and re-implementation of AB models, impairing reproducibility by a third party. Therefore, there is a trade-off between flexibility and standardization of the ABM methodology, which might be why AB models have not been disseminated as expected. Open-science practices (Muñoz-Tamayo et al., 2022) could help with the advancements and deployment of AB models, given the high levels of detail needed to program the agents.

Paradigm hybridization

Integrating different modeling paradigms (i.e., hybridization) can potentially assist in building more accurate and computationally efficient models for simulation and predictive needs.

Discrete-events modeling

Some DEM applications have been developed alone or in combination with other modeling paradigms (e.g., AB modeling) to assist animal production. The hybrid DE + AB modeling is more likely to exist because both paradigms are based on stochasticity to drive the simulation process. Such an example is the DE + AB model developed to simulate productive parameters and management of grazing sheep for meat production (Reijers et al., 2019) and the DEM + EBM for grassland-based beef cattle management (Martin et al., 2011).

System dynamics and agent-based modeling

The SD methodology, for instance, uses a "top-down" approach with a higher level of aggregation (Macal, 2010), though critical variables are necessary to mimic the intended relationships between variables and their reference mode over time. The AB methodology is preferred for building models to study problems, systems, or processes that lack central coordination (Macy and Willer, 2002). Thus, it uses a "bottom-up" approach (Macal, 2010), though more abstract simulations can also be conducted if agents are broad enough and few details are used to discover their collective behavior patterns. There is a clear separation between these paradigms in terms of model construction, but their application may overlap. AB modeling can yield similar behavior to SD models as long as the granularity increases, i.e., many more agents are added to the environmental modeling. However, this would exponentially increase its computational costs, which might limit other operational analyses of the model, such as sensitivity.

The issue is not whether an AB model can be converted to an SD model or vice-versa. The differences between SD and AB modeling are inherently rooted in how to go about modeling, and these two approaches have different concepts of building models. The SD methodology is related to the model structure (i.e., which variables are important and how they are interconnected) that creates the observed pattern, and it is also strongly connected to the feedback information that creates the dynamism in the model. On the other hand, the AB methodology stems from the idea that random relationships among agents can trigger specific responses that will generate behaviors. A critical difference is that SD uses behavior as a critical step in determining the model structure, whereas AB uses agent characteristics to create the behavior. SD uses the behavior as a cue to what lies beneath the model equations, and it must know the behavior, whereas AB does not; it is built from properties and relationships among agents without necessarily knowing the intended behavior.

The epistemology of AB and SD modeling requires a profound understanding of the differences between complexity theory and systems theory, and their heritage comprises an extensive body of literature. System theorists typically use SD modeling to identify relationships between variables within a boundary and optimize the system's output, whereas complexity theorists base their belief that complex behavior arises from many agents and their [random] interactions within the systems (Phelan, 1999). SD modeling can create complex, detailed models though this is not the main scope of its methodology, which clearly states the simplest structure that can represent/mimic the observed behavior. The question becomes, would the gain in understanding the system's behavior (or problem) offset the acquired complexity? Despite their intrinsic and profound differences, there has been an increasing consensus about combining AB and SD modeling (Phelan, 1999). Nevertheless, the question is how to draw the boundaries between systems theory (SD modeling) and complexity theory (AB modeling). The hybrid AB + SD modeling framework has been used in the energy and transport system (Shafiei et al., 2013), health systems (Cassidy et al., 2019), and many more fields (Nava Guerrero et al., 2016).

Hybrid intelligent and mechanistic or agent-based models

Although concept-driven (i.e., mechanistic) and data-driven (i.e., machine learning) are far along on their own development turf, hybrid intelligent and mechanistic model or intelligent mechanistic model and hybrid intelligent and AB model or intelligent AB model (**iABM**) are in their infancy, and protocols and algorithms are still being engineered to allow for their prime time release, especially for practical applications within the sustainable development context (Tedeschi, 2019; Tedeschi, 2022b; Vittadello and Stumpf, 2022).

However, given the surge of sensors and other instruments or technologies (e.g., Internet of Things) (Tedeschi et al., 2021), more numerical data have been collected than we have had time to develop mathematical models to connect them (in a sensible, conceptual way). The delay between mathematical model development and big data gathering is likely one of the reasons that AI has become a sensation. In some cases, scientists have not caught up with data availability by developing mechanistic models to make sense of the data. The question then becomes, are modelers and data analysts delegating their job of creating applicable mathematical models to a computer algorithm to develop a "neural representation" of the data? (Tedeschi, 2022a). While AI technology ploughs through piles and piles of data in developing models using machine learning or deep learning algorithms (a kind of modern data mining), mechanistic modeling depends on [dynamic] hypotheses for causal relationships obtained through experimental observations of the phenomenon of interest. Therefore, by design, the model-building process using mechanistic modeling is inherently more meticulous and slower than AI approaches, but it can result in additional learning experiences stemming from the model-building process.

In the fields of biomedical and clinical sciences, Baker et al. (2018) reported that 90% of the world's data were collected in the last five years (2012–2017), and only a tiny fraction of the data was used to develop causality-driven mathematical models, the remaining, a much more significant fraction of the data was geared toward supporting the development of AI-based models. Likewise, this data gathering and usage between mechanistic- and AI-based model development reflects the pattern of other fields of science, including agriculture. As discussed above, the development pace of AI-based models is much greater than that of causality-based models. The question left unanswered is, why bother developing causality-based models when AI-based models have adequate (often superior) predictability (Baker et al., 2018)? Two viewpoints seem relevant in helping to address this inquiry. The first viewpoint is that AI does not explain the reasoning behind its prediction; thus, no new knowledge is gained by using it directly, except for *improved* predictability (Tedeschi, 2019). However, through methodical and extensive simulation protocols, one might be able to learn from AI simulations. Only mechanistic, i.e., causality-based models, through their building process, can spawn *new* learning and understanding. The second viewpoint is the incessant quest for more data when AI fails. This continuous gathering of big data to satisfy AI's hunger for data creates a mutual dependency between AI and big data that might be neither sustainable nor inconsequent (Tedeschi, 2022b), possibly leading to the fracture (or even the demise) of AI technology as currently known. Modelers should employ AI technology to help build better mathematical models using hybridization. Hybridization with AI seems a natural way to move forward in building boundless and powerful mathematical models, but describing the underlying mechanisms is still critical for understanding the observed behaviors.

The ABM paradigm might be a natural way to incorporate machine learning to help agents to make decisions under exceptional circumstances. Brearcliffe and Crooks (2021) combined machine learning into an AB model to simulate the capture and metabolism of renewable resources (e.g., sugar). Although hybridization was possible, the hybrid model (i.e., iABM) did not provide the best results. Such iABM might require further advancements before it can be used for more holistic approaches, such as animal-plant-soil interactions within diverse ecosystems (Tedeschi, 2022b). Although progress has been made, more time and additional work might be required to understand the ideal applicability of intelligent systems to assist with sustainable animal production.

Reflections on building mathematical models

Upon reflecting on the usefulness of mathematical models as powerful decision-support tools in animal agriculture (or in agricultural and biological sciences in general) and the many existing modeling paradigms and classifications accessible in building effective models, one could summarize the following. "Mathematical models can be useful tools for understanding and predicting the environmental impacts of animal production systems. These models allow us to make informed decisions about how to manage resources and mitigate potential risks to the environment. One major benefit of mathematical models in animal production is their ability to provide insights into the relationships between different variables and how they may affect the environment. For example, a model might be used to understand the relationship between feed intake, manure production, and greenhouse gas emissions in livestock. By inputting different values for these variables, we can better understand how they may interact and impact the environment. Another advantage of mathematical models is their ability to make predictions about the future. By using data from past animal production systems, we can develop models that can forecast future conditions and outcomes. This can help us anticipate potential environmental impacts and take proactive steps to mitigate them. In addition, mathematical models can help us identify and optimize key factors that influence animal production, such as feed efficiency and disease resistance, which can lead to more sustainable and environmentally friendly production practices. Furthermore, by using mathematical models, we can better understand the trade-offs and potential unintended consequences of different management decisions. For example, a model might be used to understand the potential impacts of changing from one feed source to another on greenhouse gas emissions and animal performance. In summary, mathematical models provide a powerful tool for understanding and predicting the environmental impacts of animal production systems. By using these models, we can make better decisions about resource management, anticipate potential risks, and optimize production for environmental sustainability, ultimately leading to more environmentally friendly animal production practices.".

On the other hand, "while mathematical models can be useful tools for understanding and predicting the environmental impacts of animal production systems, there are also some valid arguments against their use. One potential disadvantage of mathematical models is that they are based on assumptions and can only provide estimates or predictions, rather than certainties. These models are often developed using data from past animal production systems, but the future is always uncertain and there may be variables that the model does not account for. This can make the predictions of these models less reliable. Another concern is that mathematical models can oversimplify complex systems, reducing them to a set of equations or variables that may not fully capture the nuances and complexity of real-world situations. This can lead to oversimplified or incomplete conclusions, which may not accurately reflect the reality of the animal production system being studied. Additionally, mathematical models can be time-consuming and costly to develop and maintain, requiring specialized expertise and resources. This can be a disadvantage for farmers and other animal production professionals who may not have access to these resources. Finally, there is the risk that relying too heavily on mathematical models may lead to a narrow focus on certain variables or metrics, such as greenhouse gas emissions or feed efficiency, at the expense of other important factors, such as animal welfare or the health of local ecosystems. In summary, while mathematical models can be useful tools for understanding and predicting the environmental impacts of animal production systems, they also have some limitations and may not always provide reliable or complete insights. It is important to consider these limitations when using these models and to supplement them with other sources of information and expertise, including a holistic and integrated approach to environmental management.".

There is no doubt that both standpoints have solid and valid arguments about developing and applying mathematical models in animal production concerning the environment. In reality, both arguments were produced by the OpenAI's chatbot, GPT-3 released on 30 November 2022 (https://chat.openai.com/chat), in response to the request, "*write an argument in favour* [or *against*] *of using mathematical models in animal production concerning the environment*." Chatbots are AI systems able to converse with humans through text or voice and understand one or more human languages by natural language processing algorithms (Adamopoulou and Moussiades, 2020b; 2020a). As discussed above, these two contrasting viewpoints confirm why models differ: researchers have unalike perceptions of reality and choose different modeling approaches, though perceptions can converge over time as deeper learning is pursued. Despite the increasing concerns about AI-based chatbots, we can learn and likely achieve tremendous *positive* advancements in mathematical modeling to be used as decision-support tools if we train AI to generate the correct model code. For instance, the chatbot GPT-3, and its newer generation GPT-4, released on 14 March 2023, are trained to recognize patterns in publicly available texts and reply to the request accordingly. It may provide completely wrong or non-sensical answers because it does not possess self-correction or judgment attributes to discern right from wrong; thus, the reasoning might be flawed. It brings into the debate, as discussed above, the first reason that models may not be effective because different researchers (bots, in this case) would model the same problem differently because the AI's response is still highly influenced by the framing of the question (or task to be performed).

A better use of chatbot GPT-3 (and GPT-4) for developing mathematical models would be asking whether (and how, and when) a variable of interest is related to another variable of interest rather than asking if *complete* models are relevant. This approach might lead to increased returns of excellence in building models by abetting our learning process. Many impediments weaken the learning process and reduce our ability to understand the structure of complex systems, including imperfect information and reasoning about the real-world, missed feedback loops and delays, confounding and ambiguous variable selection, little scientific reasoning (Sterman, 2000), and erroneous preconceived understanding of reality. Modelers often use pre-existing mathematical models to build (perhaps, re-invent) *new* models. Progressively speaking, using pre-established archetypes (i.e., structures that replicate known behaviors) rather than pre-existing models would be preferable because the human mind's creativity and innovative talents will likely result in improved models. In this case, AI chatbots (perhaps, *codebots*) could be employed to execute the arduous task of writing the computer code, though humans still have to *validate* the code. There is nothing wrong with existing models, but they are crammed with preconceived assumptions and, perhaps, relationships based on limited data—perfectly acceptable and correct during the existing models' conception period that could be frail for modern needs. It is worth recalling the mid-1980s fallout of the release of chlorofluorocarbon compounds into the atmosphere and the subsequent impact on the ozone layer (Farman et al., 1985). The now well-known shortcomings of the US National Aeronautics and Space Administration's Nimbus 7 satellite in confirming the actual thinning in the ozone layer due to a flaw in the computer programming to reject very low ozone readings, given an assumption at that time that instrument's reading failure could cause it (Meadows et al., 1992, pp. 151-152). By design, the US National Aeronautics and Space Administration's computer model often ignored (i.e., never reported) very low ozone concentrations. The mathematical model was eventually dealt with after scientists addressed the political ineptness and fixed misperceptions and incorrect concepts in the computer program (Meadows et al., 2004).

Mathematical models are great decision-support tools for biological systems because they help us to identify gaps in scientific knowledge, and failures are part of the model-building process. As a result, unintended consequences occur because modelers fail to understand the limitations of the mathematical model and push the boundaries, trying to solve unconventional problems (Tedeschi, 2019). How far can AI assist with model building, and, perhaps more importantly, can AI be an integral part of the model? In other words, can agents from an AB + SD hybrid model learn, make inferences, and make decisions? The main limitation with *making decisions* is how AI can assess the risk of being wrong and how AI can be penalized for being wrong. In the non-virtual world, business people and agriculture producers are penalized for making mistakes through financial means (i.e., profits). Nevertheless, how to go about penalizing AI?

These are incredible times to learn about and improve our modeling skills because AI could become the next technological wave in mathematical modeling to enhance our predictive analytics through hybrid data-centred and concept-centred modeling (Tedeschi, 2019). As discussed above, on the one hand, AI can not only be an integral part of mathematical modeling through hybridization, either using SD or AB (Brearcliffe and Crooks, 2021; Tedeschi, 2022b; Vittadello and Stumpf, 2022), but on the other hand, it can write its own code to solve specific problems (Li et al., 2022). It is expected that in the beginning, it will likely be simple programming codes for dedicated tasks, but it could expand to more complicated codes, even including SD or AB (more likely) modeling paradigms with many variables and relationships. The exciting part is that it could even fix and re-generate its own code after thorough evaluation, under supervision or unsupervised. The breakthrough for making it happen has already been uncovered for processing images, video, speech and audio through the use of the backpropagation algorithm to instruct AI to change its internal parameters of the layers (LeCun et al., 2015).

Summary

Existing modeling paradigms may share familiar developmental programming contexts and styles, including *optimization* (linear vs non-linear programming), *application* (descriptive or elucidative vs predictive or prescriptive), time representation (static or steady-state vs dynamic), time continuity (discrete vs continuous), *calculation mode* (deterministic vs stochastic or probabilistic), *nature* (empirical vs mechanistic or theoretical or rational), or *space* (homogeneous vs heterogeneous). However, different problems (or systems) might benefit from specific modeling paradigms besides being easier to codify and simulate to gain knowledge. If a variable is deemed necessary in a mental model, but its value (or unit) is unknown, assigning zero; thus, omitting it from the model is the only incorrect value it can assume. If there is no limitation on the number of variables a model can have, handicapping a model by limiting the number of variables is unacceptable, except for computational limitations. Thus, scientific knowledge progress is restrained when vital variables from a model are removed because of the modeler's perception that data do not exist. Furthermore, such an act may not only delay scientific progress; it might even act as a beacon to the scientific community in the wrong direction of science, leading to an even worse position—further away from sustainable development goals. Moreover, the delay in developing mathematical models to use big data is likely the main reason that some scientists prefer delegating their job of creating applicable mathematical models to a computer algorithm to develop a "neural representation" of the data when in reality, modelers should be making better use of the AI technology to help build better mathematical models through hybridization. Some modelers prefer

ABM because its ontology is more analogous to agricultural systems, making it a natural approach for complex systems, but combining different paradigms (including AI) is also possible and sometimes recommended. Hybridization with AI is believed to be boundless and powerful, but mental models are still needed to understand the underlying mechanisms of many animal production systems. Mathematical models will more likely help us achieve sustainable development goals if we combine concept-driven (scientific knowledge about a subject) with data-driven approaches.

Ethics approval

Not applicable.

Data and model availability statement

Data and models are not deposited in an official repository. No new datasets were created.

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